Flexible geometry:
the topology of an entangled ring puzzle

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Abstract

   This article introduces the weird world of topology using a coin and string puzzle. The puzzle is based on the 'African ball', and its mathematical structure is explained. Human levitation and entanglement puzzles involve many uses of topology, and the author believes these kinds of wonderful mathematical ideas may be put to good use in society and government. Presenting students with familiar and appealing educational resources is also important in the teaching of mathematics.

Keywords : topology, geometry, educational material, African ball

1.   The 5 yen puzzle

   A featured article by Toshiyuki Nishimori appearing beside my own in a certain publication caught my eye, and lead me to take up the challenge of the 5 yen puzzle [1]. As shown in Figure 1(1), two 5 yen coins (5 yen coins have a hole in them) are separated on the left and right. The question is whether the coin on the left can be transferred to the right to bring the coins together (Figure 1(2)). The puzzle can be easily constructed—the only materials needed are a rectangular board with three holes in it, two 5 yen coins (or metal washers), and a single piece of elastic cord. The elastic cord is fed through the holes in the manner shown in the figure. The cord can move freely through the holes, but the coin is larger than the holes and cannot be passed through them. Can this puzzle be solved? At first glance it looks impossible, but there is a surprising solution.
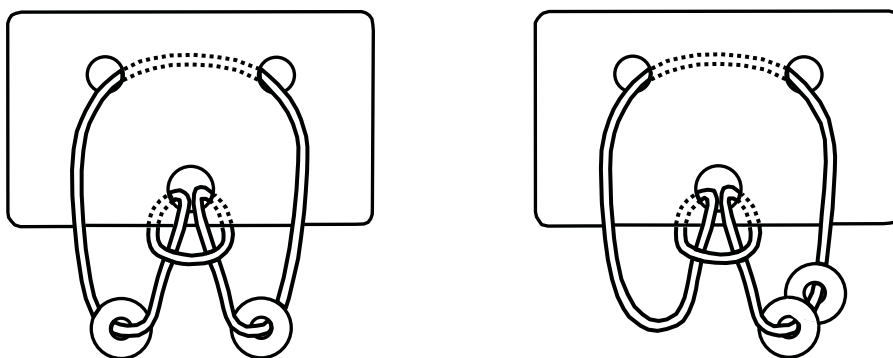
(1)   The 5 yen coins separated to the left and right   (2)   The left coin moved to the right

Figure 1.   The 5 yen puzzle (reproduced from [1], p14)

Nishimori introduced this puzzle as an event activity at RiSuPia which was held at the Tokyo Panasonic Center in March 2007. The children who solved the puzzle by themselves, and also those who could not but understood when they heard the explanation, all enjoyed themselves, and upon hearing that the puzzle is such a rare and outstanding thing, I was also inspired and decided to make one for myself.

With my immense curiosity, once hit by a thought, I must immediately put it into action. It was around 10 pm when I read the article but I did not have a board or elastic cord to hand, so for the time being I cut some holes in a piece of thick drawing paper with a pair of scissors and threaded a piece of string through a pair of 5 yen coins to construct something like that shown in Figure 1. After some time manipulating the device I had constructed, the string became tangled and the paper was reduced to shreds. At that point I remembered that in my university office I still had the materials and tools I had used to construct a boomerang. I persuaded myself to make a proper job of constructing the puzzle, and at last gave up for the night.

It was surprisingly easy to construct the puzzle. I used 3mm plywood for the board, choosing a height of 8cm and a width of 12cm and cutting it with a jigsaw. To make the holes, I first used a cutter to open a small hole, which was easy because the board was 3mm thick. I then removed the blade from the jigsaw and used it to broaden the small holes in the board, and finally I pushed a large screwdriver through and rotated it around to open up nice clean holes with a diameter of 5mm. As for the string, the explanation describes elastic cord, but thick string is also suitable.

The board and string used to make the puzzle are robust, so there is no need to worry about it breaking while being manipulated. I tried all kinds of manipulations on the puzzle, but I couldn't find the answer at all. Then it occurred to me that the solution to the puzzle might be found on the internet, and I investigated using my computer. There was even a blog article whose author had been struggling for close to a month. I also discovered that another puzzle with the same structure as the 5 yen puzzle was on sale. It is called *goen ga attara* (by Nobuyuki Yoshiagahara).

There is a DIY store on the way back to my house, and I bought myself a *goen ga attara* puzzle. I thought that this would provide the solution to the puzzle, but it wasn't to be. There was no solution included. This is only to be expected: the point is not to be told the solution, but to enjoy the process of working it out. If the manufacturer revealed the solution it would merely rob the puzzle of its pleasure. Even so, I wanted to know the solution as soon as possible. What should I do? Inside the case there was a ticket stating that if you gave up, you could post it and they would post back the solution. Recoiling from this suggestion, I reaffirmed my will to solve the puzzle by myself no matter how many days it took.

During the many manipulations I attempted with the puzzle, I managed just once to move the coin to the right hand side. I had no idea what procedure led to that result, nor could I return the coin to its original position. At that point I returned to the internet. I found something resembling a solution, though it was hard to tell and I couldn't really understand it. There was some information which included a diagram, and with that as a hint, I finally managed to solve the puzzle myself. This puzzle is really interesting, so if the reader is interested please make one and try it out.

2. The African ball

When was the 5 yen puzzle devised, and by whom? In the second chapter of Hisayoshi Akiyama's 'Book of *chie no wa*' (Entanglement Puzzles) there is a section describing 'Exemplary *chie no wa* - Top ten masterpieces'. The 5 yen puzzle is ranked second under the name 'African ball' [2], and it is revealed that many differently named variations of the puzzle are enjoyed in various countries around the world.

The puzzle is described in Pacioli's book *De Viribus Quantitatis* (*ca*. 1500), where it is referred to as 'Solomon's seal'. The history of topology begins with Euler (1707–1783), but the existence of topological puzzles prior to topology itself is indeed fascinating.

It seems that the puzzle was familiar to many people in Europe and the United States judging by its mention in various documents between 1700 and 1900, but the name 'African ball' does not appear. There are explanations according to which the name relates to the puzzle's similarity to the yoke used by native Africans to transport luggage, or the collars and shackles used to restrain unfortunate Negro slaves and prevent them from escaping or resisting. At one time a certain version of the puzzle was in broad circulation, which featured a metal figure of a Negro and was known as 'The Jolly Nigger Puzzle'. Akiyama speculates that this is the origin of the name 'African ball'.

In Japan, during the *Tenpo* stage of the late *Edo* period (1830-1844), string entanglement puzzles were known familiarly as *chie no ito* (threads of wisdom). Among these, there was a puzzle known as *shinobi no chiegoshi* which included dolls of the two traditional Japanese lovers *Osome* and *Hisamatsu* attached to a string passed through two rings. This has the same principle as the African ball, and the romantic transposition is typically Japanese.

In chapter 4 of the 'Book of *chie no wa*', there is a section describing 'Four types of entanglement puzzle classified by structure'. The African ball is classified as a component-shifting type, and its principles are explained using topological methods.

As was shown in Figure 1, the components of the 5 yen puzzle are a board, elastic cord, and two 5 yen coins, although the puzzle is the same whether one or two coins are used. Assuming that only one coin is used, the puzzle can be reconstructed with the following model (Figure 2).

(1)   The hole in the coin is traversed by the ring of elastic cord (the wide ring represents the board).
(2)   The elastic cord is looped through the wide ring.
(3)   The elastic cord is attached with a cow hitch (*a.k.a.* baggage tag loop).
(4)   The elastic cord is placed alongside the wide ring.
(5)   The wide ring is deformed with the central part as a hole.
(6)   The elastic cord is passed through both ends of the wide ring. If both ends are then closed, this forms the African ball.

Since the wide ring is closed, the coin is no longer able to move back and forth freely. It must somehow get around the part with the cow hitch. This represents the structural principles of the African ball, but it is not a solution to the puzzle.
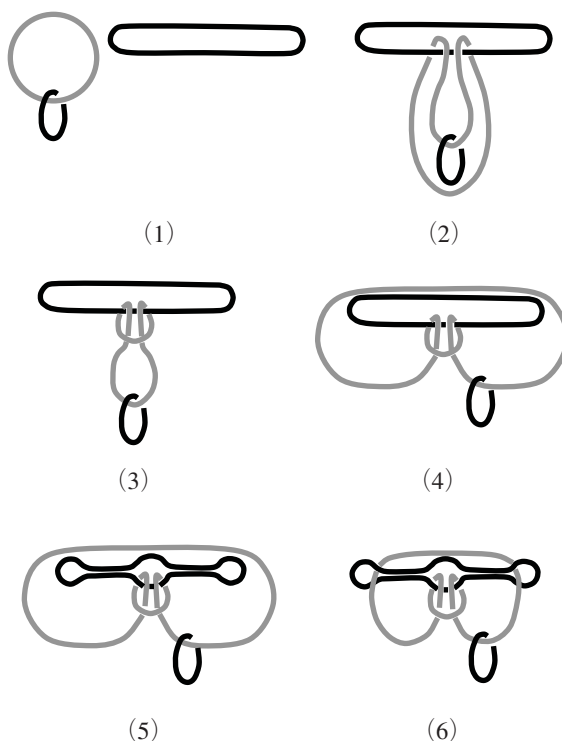
(1)   (2)

(3)   (4)

(5)   (6)

Figure 2.　Structural principles of the African ball（reproduced from ［2］, p63）

3.　Human levitation

While enjoying the 5 yen puzzle, I remembered about human levitation, which I investigated in the 1980s ［3］. There are various methods for performing human levitation, but there were some comedians—a duo who appeared on television frequently during the 1970s—which revealed the method used by Napoleons troops.

One person lies face up, and after another person has covered them with a cloth, the person laying down appears to float upwards into the air. The laying of the head and feet onto two chairs was shown to the audience, but after the cloth was placed on top the legs and body were replaced with fakes, so this trick drew the attention towards the neck. The neck and body are generally assumed to be connected in a straight line, but the neck can bend surprisingly far, to almost 90 degrees. Even when standing upright the neck can be bent backwards to almost 90 degrees, so lifting up a fake body horizontally creates the illusion of levitating into thin air. People were innocently entertained by this kind of trick during the good old days of analogue television back in the 1970s.

The revelation of this method demonstrated that it is superb technique for human levitation. There are also methods introduced by professional magicians who do not reveal their techniques, including those of Jean Eugène Robert-Houdin （France, 1805‒1871） and Harry Keller （America, 1849‒1922）. The secrets of these techniques have since been revealed and

audiences have completely lost interest, but they are significant in terms of the history of stage magic, so let's look again at how they were performed.

The so-called father of modern magic Robert-Houdin was famed for his 'floating boy' illusion in which he caused a boy to levitate up from the ground by merely touching his right arm lightly using a bamboo cane. Various levitation techniques were subsequently devised, but it was Keller who perfected a technique of the highest order in stage illusion. The characteristic feature of this illusion was the use of a metal hoop, which was passed around the levitated person's feet up to their head, then passed in front of the performer, and then, as if to make doubly sure, once again from the levitated person's feet and past their head (Figure 3). This method induces the belief that there is absolutely no support, and thus exceeds Robert-Houdin's method. It is said that it took Keller 15 years to devise the technique.
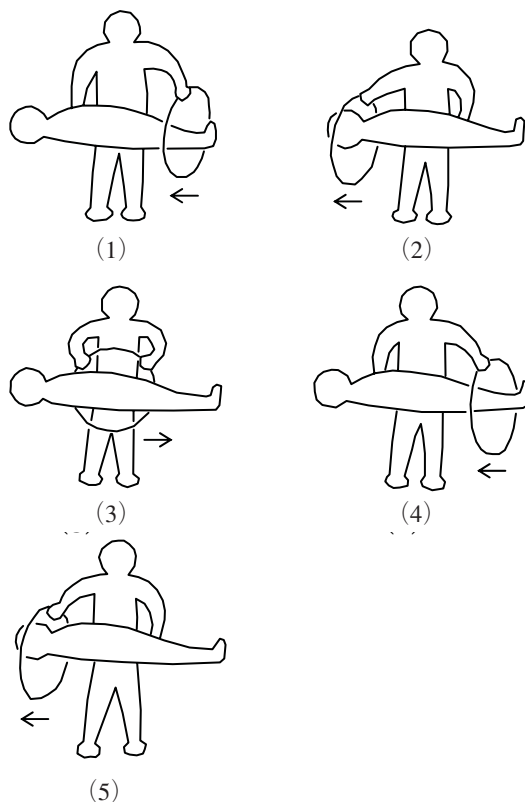

Figure 3. The hoop is passed twice to complete the illusion

The technique behind the passing of the hoop has recently been revealed by sources such as Mr. Maric, and images have been published on the web. This is unfortunate for fans of magic, but even with its secret revealed the peculiarity of the trick still surprises. There is nothing unusual about the metal hoop, the subject is not suspended from above using piano wires, nor are there superconducting magnets or the like used to lift the subject from below by means of a powerful magnetic field.

As a hint, it is essential to pass the hoop twice. The hoop is not passed for a second time in order to make extra sure; rather, it is because the illusion cannot otherwise be completed. This is also an application of topology. This illusion is not possible if the magician and the subject are directly connected, but if the magician and the subject are attached with an extended frame as shown in Figure 4 it becomes possible.
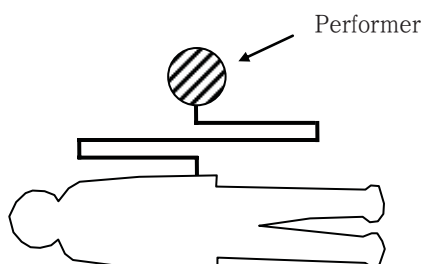

Figure 4.   Connection with an extended frame

4.   Topological entanglement puzzle

On Shiro Seyama's home page there is an interesting page entitled 'Topological entanglement puzzle'. It involves a rubber band attached to a two-holed doughnut like that shown in Figure 5, which is somehow freed from one of the holes. There is a famous topological cliché that a coffee cup and a doughnut are identical, and these kinds of puzzle may be regarded as consequences of that fact. According to Seyama, he learned about this model from Hisao Shimomachi at a national meeting of the Association of Mathematical Instruction. It is so counter-intuitive and interesting that I also referred to his home-page and made my own model using *papier-mâché*.

Figure 5.   Can the rubber band be liberated?

Figure 6 shows the solution. In the model on the top left, the rubber band is entangled with both holes, but in the model on the bottom right the rubber band is only entangled with one hole. Of course, neither the rubber band nor the doughnut has been cut. If hard metallic materials are used, cutting and so on are not possible. If the two-holed doughnut is created using

malleable *papier-mâché*, the rubber band can be liberated using the procedure shown in the figure.

The procedure has 9 steps (advancing from left-to-right then row-by-row in the figure). First, the two holes are enlarged, and what was not originally a hole becomes a hole, and what was originally a hole ceases to be a hole. By performing these manipulations, the rubber band is moved to a single hole.

Clearly, something strange has occurred. Even knowing the reason, it remains peculiar. I wanted to experiment with the model, and for the first time in decades bought myself some *papier-mâché* to make it with. When I revealed the completed model to my seminar students, they responded with the new question 'can the rubber band also be liberated from the single hole?' Do you think the rubber band can be liberated from the model shown at the bottom right in Figure 5? If you are interested, please consider this question!

Figure 6.   Completing the transfer in 9 steps

Now then, a practical question. Two rings are interlinked as shown in Figure 7. Can these two links be disentangled, as shown on the right? Since *papier-mâché* is malleable and can be freely stretched and compressed, it is possible. In case you just can't figure out how, please refer to [4], p.189. The theme this time was realizing just how far fixed ideas can be set aside: understanding flexible geometry requires a flexible mind, and these kinds of wonderful mathematical ideas are not limited to mathematics: I believe they may be put to good use in society and in government.

Figure 7.   Practical question（can the loops be separated?）

References

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[4]   Akihiro Nozaki *et al., Understanding diagrams and spaces*, Tokyo: Beret Publishing (2003) 189. (*in Japanese*)