Mathematics Column (15)

Same Birthdays

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Whenever I teach probability, I always bring up the subject of birthdays. In particular, I discuss the probability that two students in the class have the same birthday. Of course, this is also frequently discussed in high-school probability courses, but many humanities majors have never heard it, perhaps because they avoided as many math classes as possible.

The problem with performing this experiment is that you need enough students in the class, so I do a quick count when I enter the classroom. If I have well over 23 students present, then I can perform the experiment with some confidence. I start out by saying something like, “There are about forty students in this class. Everyone, tell me whether you think there are two students with the same birthday,” then tally the “Yes” and “No” guesses.

Considering that there are about 40 students in the class and 365 days in a year, most students calculate the probability as 40/365, or roughly around 10%, and thus say “No.” By this line of reasoning, you would need 366 students in the class to get a “Yes” answer, I suppose. In actuality the 10% probability that students are calculating is the chance of another student sharing their own birthday, which isn’t what the problem asks.

A couple of decades ago, when computers weren’t quite so prevalent, I would have students individually announce their birthdays and raise a hand when a match was made. With a classroom of forty it is easy for students to mishear reported birthdays, however, so now I have students email me their birthdays, which I record in a spreadsheet. Students can watch me do this on my monitor, and always get very excited when we find a match. They are truly mystified when we find two pairs.

After doing this, I explain calculating probabilities from complementary events. We calculate the probability that all forty students in the class have different birthdays, and from this the probability that at least two share a birthday. Assuming a 365-day year, the first student can select 365 out of 365 days, so the probability is 365/365. The second student can then select from 364 days—any day other than the one the first student selected—so the probability is 364/365. Continuing this on to the fortieth student we get a probability of 326/365, and multiplying this with each of the previous calculations gives 0.109.

The complement of this, which represents the probability of a match, is 0.891. Going from the second to the fortieth person gives individual probabilities of 0.997–0.893, all of which are fairly close to 1, so it is surprising that multiplying them together gives 0.109.

Investigating the number of students n needed to reach the point where the probability of a match exceeds 0.5, we can see that the number is $n = 23$. In other words, if there are at least 23 students in the class, a match is more likely than no match. It is interesting that this 23 value is less than 10% of the number of days in a year.

The complementary approach to probabilities is highly effective. The complement to “all birthdays are different” is “at least two birthdays are the same,” so the relation here is “never” vs. “at least one.”

In one experiment with forty students, not only did we find two pairs of students with the same birthday, but one pair were sitting next to each other. In another experiment with 60 students a few years ago, we found three pairs, again with one pair sitting adjacent. Apparently sitting next to your match is another probability that is higher than expected.