Mathematics Column (21)

Penney's Game

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I would like to introduce you to an interesting probability game that uses standard playing cards. As you know, a standard deck of cards has four suits—spades, clubs, diamonds, and hearts—each with thirteen ranks, from Ace (A) to King (K), for a total of 52 cards. Spades and clubs are black, diamonds and hearts are red, so there are 26 red (R) cards and 26 black (B) cards. The game I will describe is for two players, and involves these two colors.

Shuffle the deck and place it face down on the table. Turn the cards over one at a time, trying to guess the color pattern of groups of three cards. For example, you might guess BBR or RBR. The winner is the player whose pattern comes up first.

So let's play a game. I'll assume that you have selected RRB as your pattern, so I will take BRR. I will use a random number generator on my computer to generate a random sequence of R and B values. Here it is:

BBRBBRRRBBBRBRBBBRBRRB-
BRBRBRRRRBBBBRRBBRR-
BRRRRRBRBB

These represent the order in which we turned the cards up, one at a time. Note that we have a winner after the seventh draw; the last three values in BBRBBRR are BRR, so I took that hand. We set those cards aside and continue playing. Here is the result of our game, with vertical bars indicating the end of a hand (the last three cards are considered a tie here):

BBRBBRR|RBBBRBRBB-
BRBRR|BBRBRBRR|RRRB|BBBRR|BBRR|
BRR|RRRB|RBB

So in the end, we had eight successful hands, with BRR (me) winning 6–2.

Now it would seem that there is an equal probability for either RRB or BRR appearing, and thus equal chances of winning, so a 3:1 victory of BRR over RRB might seem to be statistically improbable. Let's say that having lost with RRB and expecting some trick, for our next game you take my BRR pattern. I in turn decide to play BBR. Here is the random sequence my computer generated for the game:

RRBRBRRBRRRBRRBBRBRBBBRRR-
BRRBBBRBBBRBBBRBRBBRRB-
BRRBR

As before, here are the results, with vertical bars separating hands:

RRBRBRR|BRR|RBRR|BBR|BRBBBR|RRB
RR|BBBR|BBBR|BBBR|BRBBR|RBBR|RBR

As you can see, we played eleven hands, and once again I win, 7–4.

Of course, I am not manipulating the data above for the sake of a good column. Indeed, it has been mathematically proven that the player making the second choice can always win, and that a case like our first game (RRB vs. BRR) should result in a 3:1 victory, while one like our second game (BRR vs. BBR) should result in a 2:1 win. The origins of this are a 1969 publication by Walter Penney, who proposed a similar game using coin flips in place of red and black cards. (Using red and black playing cards without replacement doesn't result in a random sequence in the strictest sense of the word, but card draws are easier to perform than coin flips, and so recommended for illustrative purposes.)

Playing a few times will likely convince you that the player making the second choice can construct a perfect game. Of course, this raises interesting questions related to how the second player determines the winning pattern and the probabilities involved. For the answers to those questions, I direct interested readers to the referenced paper.

Ref

Pattern Matching Probabilities and Paradoxes as a New Variation on Penney's Coin Game
http://www.ijpam.eu/contents/2010-59-3/10/10.pdf