The Mathematics of Egg Shape

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Abstract: This paper explains why the shapes of eggs are oval, and why eggs stop on slopes. After touching upon Descartes' and Cassini's oval curves, eggs are classified into 4 groups: oval, pyriform, circular, and elliptical. Because only oval and pyriform eggs stop on slopes, it is explained that egg shape may be related to Darwin's theory of evolution.

Keywords: Oval, Descartes' oval, Cassini's oval, Circle, Ellipse, Darwin's Theory of Evolution

1 Eggs stop on slopes

The main topic of interest in biology at the moment is DNA, but if you pay attention to shapes and numbers you will see that many mathematical elements are included, and in this essay, I'd like to explain the peculiar shape of eggs. I made a presentation about this topic at the August 1979 'mathematical seminar' under the title 'The shape of eggs', which was a long time ago but rereading it now, their shape seems just as strange (see Nishiyama, 1986).

Hasn't everyone thought that the shape of eggs used for cooking is strange? Eggs are oval, which means egg-shaped, so while the inquiry is just like a Zen riddle, let's reveal the secret little by little. In mathematics, a representative example of round forms is the circle. Circles may be determined by their center and radius. At high school we learn about ellipses, which are circles stretched sideways and their curve is determined as a constant sum of the distance to two foci (or fixed points).

Eggs are neither circular nor elliptical. Eggs are oval. If you observe an egg closely, the distance from the center is not a fixed circle. The horizontal aspect has a longer ellipse-like form. Observing closely once again, one horizontal direction is roundly curved but the other is pointed (Figure 1). This is the shape of an egg. The longer axis is called the major axis, and the shorter is called the minor axis. The rounded end is called the base and the pointed end is called the tip. Since eggs are actually three dimensional bodies, they should not be expressed in terms of circles or ellipses but rather spheres and ellipsoids. However, it is sufficient to think in terms of cross sections, so I will explain here using planar shapes.

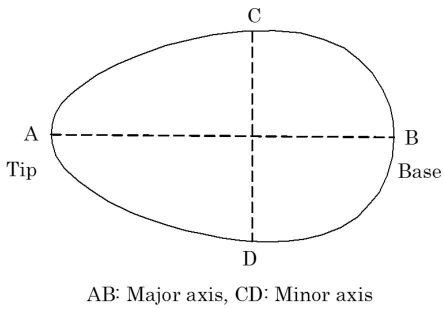

Figure 1: the shape of an egg

Let's think about why eggs are shaped as they are. Place an egg on a tabletop. The major axis is not parallel to the surface of the table. The pointed end is closer to the surface, and it stabilizes with the rounded end further from the surface (Figure 2). For an ellipse the major axis would be parallel to the surface. If we consider the egg's center of mass we can understand why it stabilizes with the major axis tilted. For circles and ellipses, the center of mass is in the exact mid-point between the vertical and horizontal axes, but for an egg, since one end is pointed and the other is rounded, the center of mass is slightly offset towards the rounded end. What happens when an egg with its center of mass offset from the center is placed on a tabletop? As shown in Figure 2, the gravity force W from the egg's center of mass O, and the reaction force N from the contact point P lie on the same straight line, and the major axis stabilizes with a tilt. I explained this using the terminology of physics, but everyone knows that eggs tilt like this.

What happens because of the tilt in an egg's major axis? If an egg is

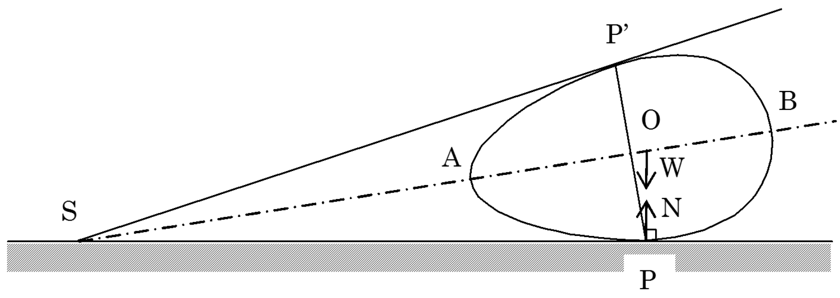

Figure 2: Eggs tilt

placed on a slope, no matter what position it is placed in, it settles in a stable position without rolling away. This is peculiar. The stable position is with the pointed end oriented towards the top of the slope, and the rounded end towards the bottom of the slope (Figure 3). I'd like for those readers who have until now had no interest in the shape of eggs to begin by confirming this experimentally.

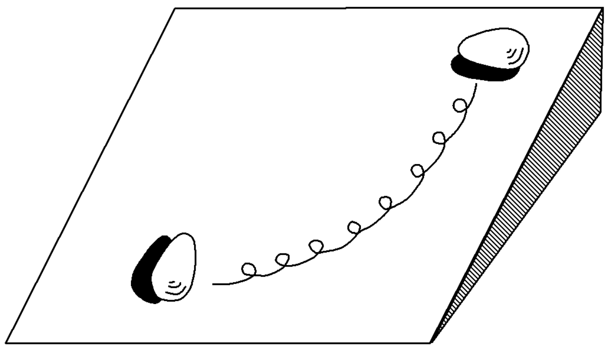

Figure 3: Eggs settle on slopes

My article, "The shape of eggs" was subsequently included in an independent book, "Why are eggs egg-shaped?" (Nihon Hyoronsha). I received only one protest regarding the article from a certain reader. An egg was placed on a tilted tabletop and rolled, but it did not stop. They proceeded to say that my claim was a lie, and ought to be revised. I became worried, and repeated the experiment, but the egg did stop after all.

So why didn't the reader's egg stop? Perhaps they angled the tabletop at about 30 degrees and then rolled the egg. The tabletop must be tilted less than 5 degrees, and it must be released carefully and gently. Figure 3 is a schematic diagram intended to show the slope, and if it is interpreted as showing a slope of 30 degrees it could cause a problem. Also, if the surface of the table is too smooth the egg might not stop. A certain degree of frictional resistance is necessary. In addition, I have also performed the experiment with a boiled egg, and sometimes it doesn't stop. A member of staff brought an egg to use for recording on a particular television program, but the egg was boiled. I was told that there is a greater risk of breakage during transportation with raw eggs, but the center of mass in the boiled egg is in a slightly different place and the egg did not stop. Also, when an egg is boiled it may cease to be rough and lose its friction. On that particular occasion the egg was exchanged for a raw egg at short notice.

It can be confirmed that paper cups stop on a slopes in the same way as eggs when they are rolled. Paper cups usually have a large circular end to drink from, and a smaller circular base. If the side surface gripped by the fingers is extended, it forms a cone. Eggs and paper cups both approximate cones (when extrapolated). The problem of eggs or paper cups rolling on slopes can be replaced with the problem of a cone rolling on a slope. It's not that hard to think about why a cone stops rolling on a slope (Figure 4).

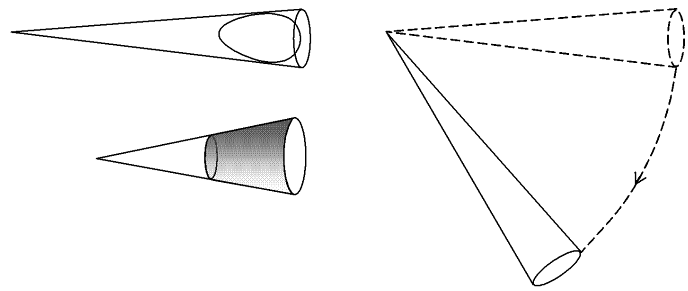

Figure 4: Eggs and paper cups approximate cones

Furthermore, eggs and paper cups also share the characteristic that, besides slopes, they do not roll far on horizontal surfaces. Suppose that an egg rolls away when a bird is cradling eggs in order to hatch chicks. The bird cannot move to bring back the egg. In the same way that a cone describes a circular arc and returns to its original position, eggs also describe circular arcs and return to their parents.

2 Descartes and Cassini's oval curves

Descartes and Cassini's methods may be used to describe oval curves. Descartes defined oval curves as follows (Descartes, 1637). Two circles form the basis.

One circle has center O_1 and radius r_1, while the other has its center O_2 offset in the x axis by a and has radius r_2. Two parallel lines are drawn, one going through each center, and the intersection point of each line with the other circle is denoted as either A or B (in this case O_1B and O_2A are parallel). The intersection point of O_1A and O_2B, P gives the coordinates of the oval. For $m, n > 0$, $m\overline{O_1P} + n\overline{O_2P}$ relates the parameters as a constant value. When $m = n$ the curve is an ellipse, and the radii of the two basis circles are equal ($r_1 = r_2$). Figure 5 was drawn using a Visual Basic program with the values $a = 1, r_1 = 1.2$ and $r_2 = 1.8$. It certainly has an egg shape.

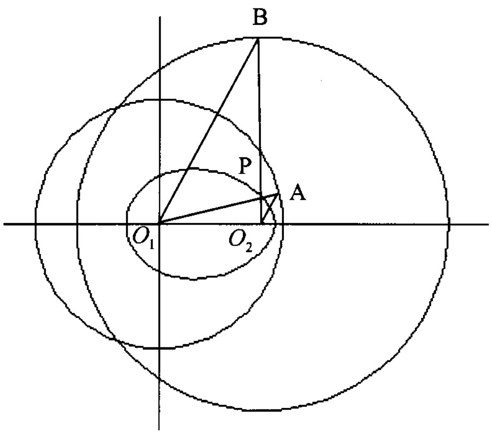

Figure 5: Descartes' oval curves

Another way to describe oval curves is Cassini's method. Cassini defined the oval curve as follows (Cassini, 1680). The trajectory of points X such that the product of the distances to two fixed points (or focii) is constant describes an oval curve (Figure 6). With O, the mid-point of A, B as the origin, and the line joining A and B as the x axis, the equations relating the orthogonal axes are as follows.

$$(x^2 + y^2)^2 - 2a^2(x^2 - y^2) = k^4 - a^4$$

Note that $\overline{AB} = 2a$ and $k^2 = \overline{AX} \times \overline{BX}$.

In particular, if $a^2 = k^2$ then O is the nodal point of the curve. In this case, the curve is known as a Lemniscate. Cassini's oval curve is expressed as an implicit function so it cannot be graphed in this form. If the substitutions $x = r\cos\theta$ and $y = r\sin\theta$ are made then the function is represented in polar coordinates by the following equation.

$$r^2 = a^2 \cos 2\theta \pm \sqrt{a^4 \cos^2 2\theta + k^4 - a^4}$$

Contour drawing software can be used to draw the oval curve specified by the former equation. The latter equation can be drawn comparatively easily using a Visual Basic program.

In the case of an ellipse, the sum of the distance from the two fixed points is constant, but in Cassini's oval curve it is the product that is constant. Several ovals were drawn with different values for the parameter k. Figure 6 shows the diagrams for $k = 1.4, 1.2, 1$ and 0.98 when $a = 1$. When $k > a$ the outer curve is an elliptical oval. As k gets smaller the region near $x = 0$ gets thinner, and when $k = a$ it becomes the Lemniscate curve with origin O. For $k < a$ the curves divide into two and it becomes oval. The most interior curve is the egg shape we have been discussing here.

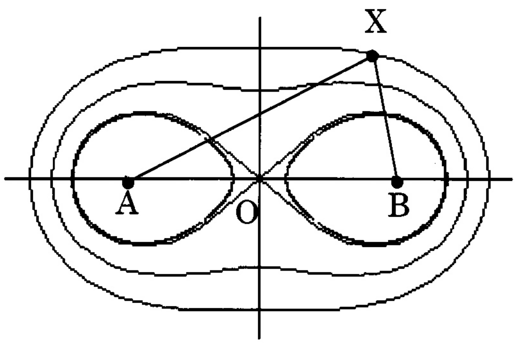

Figure 6: Cassini's oval curves

Cassini (1625-1712) was an Italian astronomer who was invited to Paris by Louis XIV and became the first director of the astronomical observatory. He made many accomplishments, including the measurement of Jupiter and Mars' periods of rotation, discovering the gap in Saturn's ring and its four moons, and calculating the distance between Mars and the Sun. Cassini thought the orbits of the planets were oval, but in fact Newton (1642-1727) showed that they are elliptical orbits with the Sun situated at their focal point. There are also some interesting properties such as with the torus (a doughnut or floating ring). If it is cut by a plane parallel to its axis of rotation, the cross section revealed is one of Cassini's ovals.

3 Eggs have various shapes

In biology, ‘egg’ refers to an egg cell, but the word is usually used to mean the thing which is laid externally. The cytoplasm includes the nutritious material of the yolk, which is surrounded by various materials such as the white of the egg. There are various sizes of egg such as the whale shark’s at 68cm×40cm, the ostrich’s at 16cm×12cm, and a type of pigeon at 1.2cm×0.8cm (I was able to see these in 2005 when I had the opportunity for overseas study in Cambridge. I used a free day to visit the museum of natural history in London, where they had these eggs on display). Paying attention to the shape of the eggs, they can be classified into 4 groups: oval, pyriform, circular, and elliptical (Figure 7).

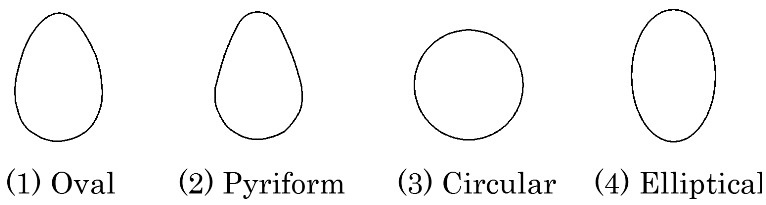

Figure 7: Types of shape

Chicken’s eggs are representative of ovals. Pyriform eggs are common among seabirds such as Auks and murrelets, and the difference between the tip and the base is larger than for an oval. Round eggs are represented by sea turtles, and ellipses by ostriches.

Chickens now lay their eggs in flat places so there is no need to worry about them rolling away and breaking, and their eggs are oval. Perhaps the eggs are oval so that when the hen is brooding and keeping her eggs warm they don’t roll far away but describe a circular arc and come back. The sea turtle lays its eggs in a sandy beach, and the ostrich lays them in grassland, both of which are flat places where there is no danger of the eggs rolling so it is understandable for the eggs to be round or elliptical. Auks and murrelets lay their eggs on narrow rocky shelves. It is common for the rock shelves to be sloped, so the reason behind their pyriform shape also makes sense. Gulls and gannets are also seabirds, but since their eggs are not pyriform there is a danger of them rolling, so they build nests. Thinking about it this way, one can understand that of course, that’s the reason why eggs are egg-shaped!

4 Darwin's evolutionary theory

Were eggs always egg-shaped? Were the shapes of eggs fixed from the beginning, or were they set later?

Darwin proposed a theory of evolution. It is known as the theory of natural selection. It was discovered in parallel by Darwin and Wallace at the same time, and is a theory about the cause of evolution. Proliferation is a principle of living things, and as a result of the consequent competition for survival, individuals with adaptations suited to their environments arise. These adaptations are transmitted to their descendants. The idea is that living things thus gradually advance in the direction of these adaptations to their environment and evolve. Darwin related this theory in full in 'The Origin of Species', and as a result the theory of evolution was widely acknowledged. Even in the 20th century it occupies a central place in the contemporary theory of evolution. 'The Origin of Species' is included in the Iwanami paperback library, so I recommend casting an eye over it.

As I will show below, egg shape may be seen as material supporting Darwin's theory of evolution. Figure 8 shows the structure of an egg.

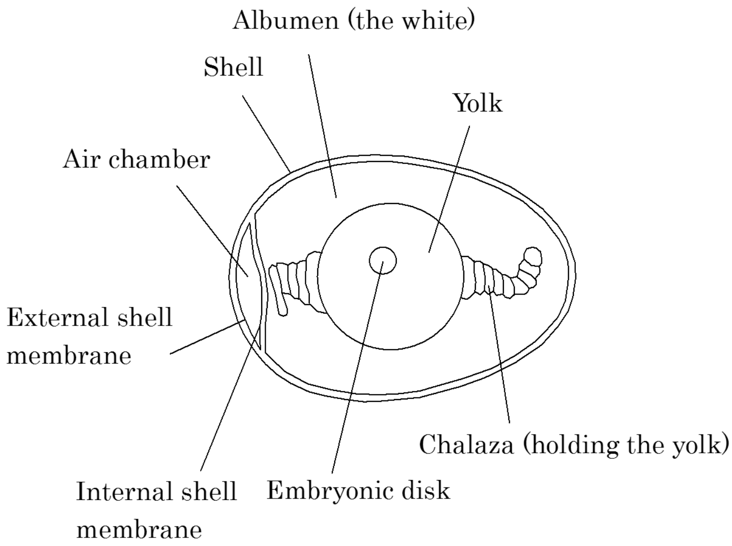

Figure 8: The structure of an egg(from the 'Picture Book of Birds', Shogakukan, p.160)

The construction of an egg includes its casing, the egg shell. Breaking this open reveals the albumen (commonly known as the white), the yolk, the chalaza which holds the yolk, the embryonic disk which is the origin of the bird's body, an air chamber, the shell membrane, and so on. Surprisingly, while chicken's eggs are certainly oval, breaking one open and removing the yolk reveals that the yolk is close to spherical. The yolk is a ball. It is slightly distorted due to the influence of gravity, but it is basically round.

Do you know about *tamahimo*, which is sometimes sold at poulterers' storefronts? It is chicken ovaries and oviducts (Figure 9). Besides eggs and chicken meat, these kinds of spoils are also put on sale. The *tama* in *tamahimo* refers to the ovaries, and the *himo* refers to the chicken's oviduct. While chickens that could no longer lay eggs were sold as meat, in the old days, the *tama* and *himo* were said to be nutritious and considered quite precious spoils.

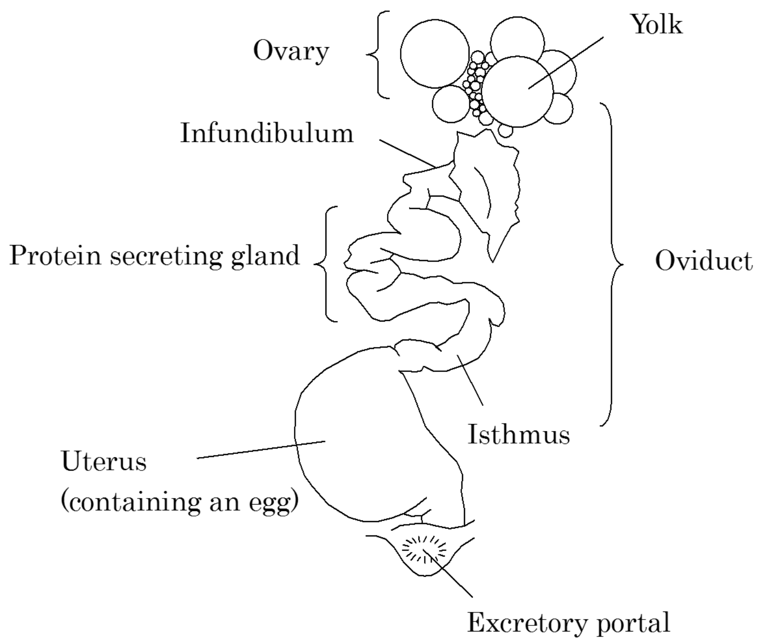

Figure 9: The ovaries and oviduct(from 'Chicken and egg', Saito Masami, Dowashunju)

Chickens lay one egg per day, but here we can examine the process behind the generation of a single egg before it is laid.

Inside the ovaries there are many yolks, like bunches of grapes. They all have an approximately spherical shape. When one is sufficiently large, it enters the oviduct via the infundibulum (funnel), and while receiving secreted proteins, the albumen and ephippium (also known as the soft shell) begin to form. The shell forms inside the uterus through the composition of calcareous material. The time between its passage into the oviduct via the infundibulum until laying is about 24 to 27 hours, and the total length of the oviduct is about 70 to 75 cm.

The shape of the shell is not determined at the moment of release from the ovary, but during the 19 to 20 hours that it is retained in the uterus. The formation of the shell in the uterus is related to dietary calcium. If there is a calcium deficiency then the shell will be thin. The yolks in the ovaries are spherical, but when excreted, the shell may be spherical, elliptical, oval, or pyriform.

If an egg laid onto a rocky shelf were round or elliptical, then it would roll and break, so this type probably died out. If the egg were, even by chance, oval or pyriform then because of its shape it would not roll and the type would be saved. From this regard, while the distance from the ovaries to the excretory portal is only a few tens of centimeters, this path can be thought of as integrating the history of evolution since before archaeopteryx (in the Jurassic period).

5 Applications of egg shape

I have reached an explanation from a biological perspective that bird's eggs have the successful property of returning to their parents without rolling and breaking, but egg shapes are not limited to biology. There have been many attempts to use it in our daily lives. Searching for the keyword 'oval' on the internet reveals some interesting pages.

There are egg-shaped sludge digestion chambers. In comparison to previous tubular digestion chambers, it is explained that they are superior in terms of water-tightness and air-tightness. There are egg-shaped gutters. The cross section of flowing water is egg-shaped, so they are said to have good drainage. There are egg-shaped car bodies. Streamlining has developed in cars with a front end corresponding to the tip of an egg and a rear end like the base of an egg.

There are egg-shaped mobile phones. In fact they are elliptical rather than oval, but they were also designed with eggs in mind. There are egg-shaped speakers. It is explained that by suppressing various oscillations and echoes occurring inside the speaker, the egg shape is used to pursue reproduction of the original sound. In addition, there are concept products such as egg-shaped refrigerators, egg-shaped washing machines, and so on. That doesn't mean to say that they have all been scientifically proven, but it can be easily understood that the shape of eggs is not only relevant to biology, it also has a significant influence in our lives.

References

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