Miura Folding: Applying Origami to Space Exploration

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Abstract

Miura folding is famous all over the world. It is an element of the ancient Japanese tradition of *origami* and reaches as far as astronautical engineering through the construction of *solar panels*. This article explains how to achieve the Miura folding, and describes its application to maps. The author also suggests in this context that nature may abhor the right angle, according to observation of the wing base of a dragonfly.

Keywords: Miura folding, Solar panels, Similar diagrams, A and B paper size standards

1. An application involving solar panels

Perhaps you are familiar with the concept of Miura folding? Miura folding has a broad meaning, and is an element of *origami*. Its applications span from ancient Japanese traditions to astronautical engineering, and since it is also of interest mathematically I'd like to introduce it here.

The 'Miura' in Miura folding is derived from the name of the man who devised it, Koryo Miura. It was some time ago, but it is understood that this folding method occurred to him when he was researching aerospace structures while enrolled at Tokyo University's Institute of Space and Aeronautical Science. Rockets launched into space make use of the Sun's energy while they fly. The devices that gather this solar energy are solar panels, but these panels cannot be opened until after the launch. The solar panels are folded down as much as possible in order to pack them into the rocket, and then after the rocket blasts into outer space they quickly unfurl. When the rocket returns to the surface of the Earth, they must be folded down and re-stowed. The idea of Miura folding was realized after thinking about how this sequence of actions could be achieved not by humans, but by robots.

Miura folding has been introduced in newspapers and magazines and so on, and detailed discussions by the authors can be found in these articles. Here I would like for everyone to experience the wonderfulness of mathematics by performing practical experiments on the principle of Miura folding, (see Asahi Newspaper, 1994, Miura, 1988, and Miura, 1993).

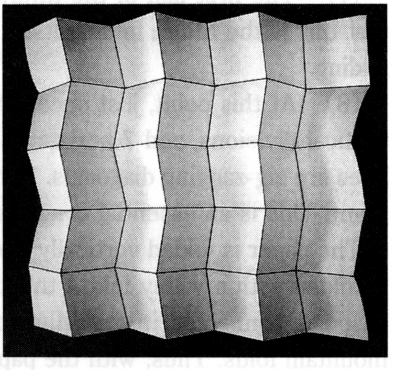


Figure 1. Miura folding

Utilizing Mirua folding in this case involves holding the bottom left and top right corners of a piece of paper with the fingers, as shown in Figure 1. Placing the paper on a desk, a single movement can be used to open and close the paper if it is pulled in diagonally opposing directions. The fold behaves as if it has been remembered so it can be described as shape-memory *origami.* This can't be done with usual maps. According to *origami* specialists, Miura apparently did not discover this type of fold himself, but recognizing the hint from ancient Japanese *origami* and applying it to solar panels was a great achievement.

2. Let's try making a Miura folding

Allow me to explain the Miura folding method following the description by Koryo Miura himself in the book 'Solar Sails', p53-66 (Miura, Koryo, 1993). I'd like the reader to attempt the folding according to Figure 2.

(1) First prepare a piece of A3 paper. B4 may also be used, but as the folds build up it gets smaller and smaller. The larger the paper the easier it is to fold, and I therefore recommend A3.

(2) Fold the paper into 5 equal vertical parts, and 7 horizontal parts. If A3 paper is used then the vertical height is 297 mm so it cannot be exactly divided into 5 parts. Since it doesn't matter whether the two ends are too short, or too long, for the time being the part right in the middle should be maintained at the same length. The five vertical divisions should alternate mountain folds and valley folds, like a concertina.

(3) Next, it is folded horizontally into 7 divisions. The objective is to apply the diagonal fold shown by the dotted line. When 7 horizontal layers have been made, the 3rd layer from the left is bent around diagonally. The diagonal fold is made such that the tips are at a ratio of about 2 to 1. Such a steep angle makes it easy to achieve the Miura fold.

(4) Next, the 1st layer is folded back along its length. At this point the initial horizontal line and the current horizontal line should be made parallel.

(5) The diagonal is bent back in the same way. Again this should be parallel to the original diagonally folded line, *i.e.*, a zig-zag should be repeated. The left and right edges of the folds should be layered exactly. Only the last tab of paper is not layered.

(6) Flip it vertically in this state. The reverse side should be folded up in the same way using a repeated zig-zag while keeping each part parallel.

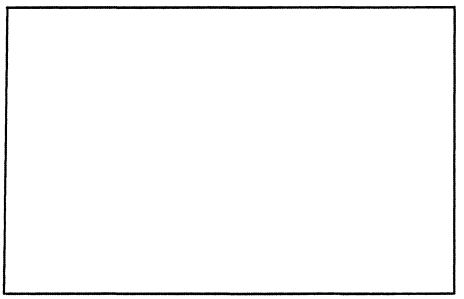
(7) The first half of the Miura folding is now complete. Some people mistakenly believe that this is the Miura folding itself, but in fact this is no more than a preparation for the Miura folding.

(8) At this point, just once, let's spread out the paper on a desk and take a look. It has 5 vertical divisions, and 7 horizontal divisions. The horizontal lines are parallel, but the vertical lines are zig-zagging diagonals. Perhaps you will notice that the smallest element is a parallelogram. This is an essential condition of Miura folding.

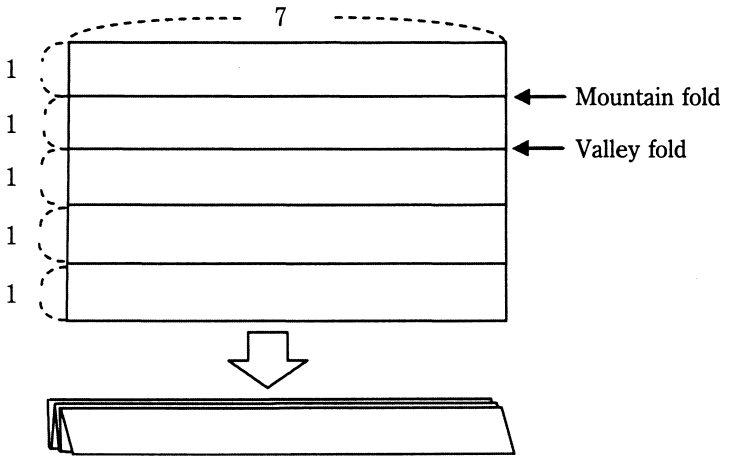
The paper is folded vertically from the edge on the left using a mountain fold. The next edge is folded with a valley fold. I think you will notice as you fold it and see, but since the whole shape is connected in the Miura folding, it cannot be achieved by repeating only individual mountain folds. Thus, with the paper gripped between the finger tips, we can stop without absolutely completing each fold. Also, if the valley folds are difficult, flipping the paper over and

using a mountain fold might be easier.

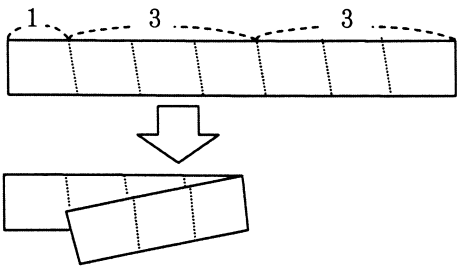
By repeating the mountain folds and valley folds following steps (9) to (13), the whole body is collapsed down to the left hand side. Since the whole body is connected in the Miura folding, collapsing down the left-right axis also collapses the vertical axis at the same time. In order to avoid damaging the folds during this process, it is important to proceed carefully. In this way, the Miura folding is completed. Everyone should confirm that the completed Miura folding can be opened and closed in a single motion like that shown in Figure 1, by pinching it between the fingers, in the bottom left and the top right corners.



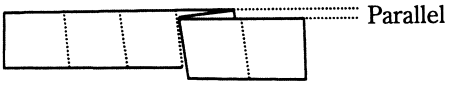
(1) Prepare a sheet of A3 or B4.



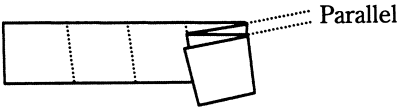
(2) Fold it into five sections vertically.



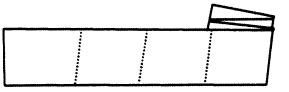
(3) Fold the length of the 3rd layer diagonally.



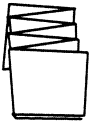
(4) Fold back the length of the 1st layer parallel to the initial line.



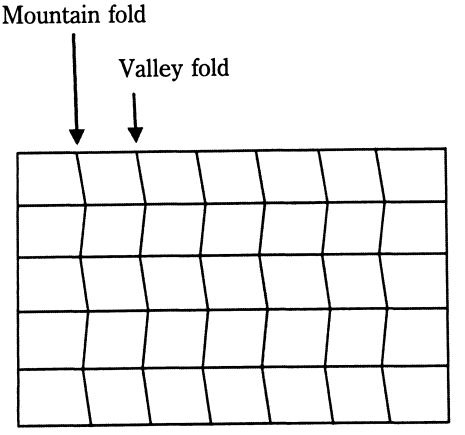
(5) Make a repeated zig-zag.



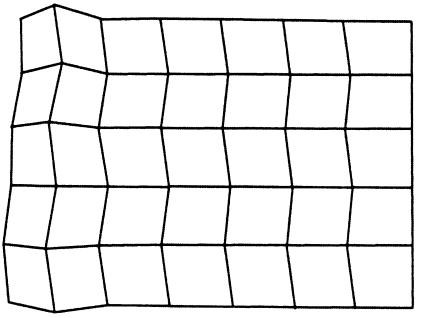
(6) Flip it vertically and fold the reverse side in the same way.



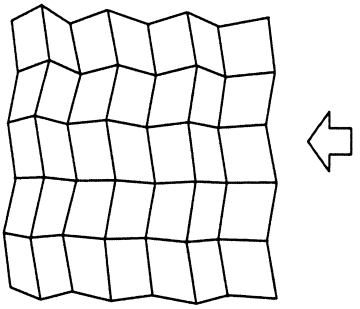
(7) At this point half of the Miura folding is now finished.



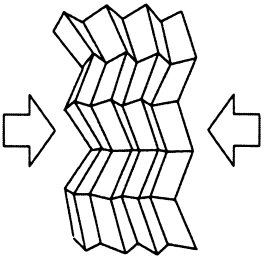
(8) Spread the paper out on a desk.



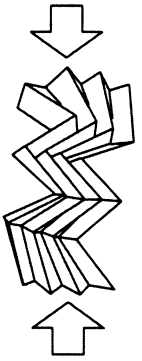
(9) Fold the leftmost column with a mountain fold, and then the next column with a valley fold.



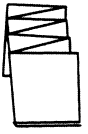
(10) Keep repeating the sequence mountain fold, valley fold, mountain fold, valley fold.



(11) Fold down the left and right.



(12) Fold down the top and bottom at the same time.



(13) The completed Miura folding.

Figure 2. The Miura folding procedure (drawn up according to reference Miura, 1993)

3. Similar diagrams

A3 or B4 paper was used for the Miura folding, with 5 vertical divisions and 7 horizontal divisions. Allow me to explain now why this size of paper and odd number of divisions were used.

According to JIS standards, paper sizes may be one of two types, the A series and the B series. The area of a piece of A0 is $1m^2$. Half this size is A1, taking half again yields A2, and so on for the A series. The area of a piece of B0 is $1.5m^2$. Half this size is B1, and half again is B2, and so on for the B series. The dimensions of the A series and the B series from 0 to 6 are shown in Table 1. The units are millimeters.

A series	No.	B series
841 × 1189	0	1030 × 1456
594 × 841	1	728 × 1030
420 × 594	2	515 × 728
297 × 420	3	364 × 515
210 × 297	4	257 × 364
148 × 210	5	182 × 257
105 × 148	6	128 × 182

Table 1. JIS Standard Paper Sizes (mm)

The interesting thing from a mathematical perspective is that the A series and the B series are both composed of similar diagrams. Since they are similar, the ratio of the vertical to the horizontal is the same for all the shapes, which are rectangles. The fact that copy paper sizes are similar diagrams should have been learned in junior high-school, but after finishing their exams, many university students and members of society completely forget about this fact. It's a shame that when you ask them to obtain the ratio of the vertical and horizontal dimensions of copy paper, most people end up unable to answer. But rather than committing Table 1 to memory, I'd like for the reader to understand the principle of similarity, and be able to assemble an equation and obtain a solution in this way.

It's possible to find the ratio of the vertical and horizontal dimensions in the following way. Let's denote the ratio of the rectangle's vertical and horizontal dimensions as 1 to x. Thinking about half of this rectangle, its vertical dimension will be $\frac{x}{2}$, and its horizontal dimension is 1, so the ratio is $1 : x = \frac{x}{2} : 1$. Solving this equation yields $x = \sqrt{2}$, *i.e.*, the ratio of the vertical and horizontal dimensions of copy paper is $1 : \sqrt{2}$, where $\sqrt{2} \approx 1.4142$.

The divisions used in the Miura folding are 5 vertical, and 7 horizontal divisions. Since the ratio of the vertical and horizontal dimensions of the largest element is $\frac{7}{5} = 1.4$, this value is close to $\sqrt{2}$. This means that the benefit that the element can be folded up with a shape close to a square can be anticipated. Also, both 5 and 7 are odd numbers of divisions. If the number of divisions is odd, then when the paper is gripped between the fingertips in the bottom left and

top right corners and pulled, the paper does not flip over, but rather it spreads out. Any number of divisions in the vertical and horizontal directions should be acceptable for a Miura folding, although the reference above has taken care to investigate all the possible configurations in this neighborhood.

4. Application to maps

Solar panels which apply the principle of Miura folding have actually been loaded onto the experimental Japanese satellite N2, and spread out in space. Owing to the principle of Miura folding they were spread in a single motion, but I heard that the closing motion in order to pack them away did not proceed well. Perhaps it is harder to close it than to open it.

Miura folding is truly wonderful. When I introduced it at a research group or symposium on mathematics education, one of the participants informed me that they had discovered a map that utilized Miura folding. It was being sold by the Kyoto tourist board. I quickly made arrangements and ordered one. The map of Kyoto city center was indeed made using Miura folding. Perhaps tourists might take out the map from an inner pocket, spread it open with a single movement, confirm their destination, then close it once again with a single movement and put it back in their pocket. Besides the Kyoto city center tourist map there was also a road map of the highways in the capital. However, the recent advancements in car navigation systems might spell the gradual disappearance of traditional paper road maps.

Well, now you've persevered with my review of Miura folding using 5 vertical and 7 horizontal divisions as explained above, let's move on. The point behind Miura folding is that the horizontal lines are parallel while the vertical lines are in a zig-zag. If the verticals and horizontals are both parallel, that is to say the vertical and horizontal lines are at right angles, then it cannot be opened and closed with a single movement in the manner of a Miura folding. I'd like for the reader to make a model with 5 and 7 divisions using normal folding, and then perform a comparative investigation with the Miura folding.

Also, the folds in maps made using Miura folding are slightly offset, with the result that they are difficult to cut like normal maps. Refer to Figure 2(13). Miura folding has parallel horizontal lines, and zig-zagging vertical lines. I wondered if it could be made with zig-zagging horizontal lines as well. This is interesting mathematically, and is possible. Attempting to confirm this by drawing up diagrams revealed that it could be folded just by allowing the positions of the parallelograms to be irregular (refer to Nishiyama, 1995). However, I'm not sure how meaningful this really is.

5. Does nature abhor the right angle?

It is said that Koryo Miura devised Miura folding in 1970 after observing the wrinkles in old people's brows and in the surface of the Earth in photographs taken from spaceships. The idea behind Miura folding was obtained through a detailed observation of nature.

The long running author Toda Morikazu of the 'Toys Seminar' in the periodical 'Mathematics Seminar', has also dealt with Miura folding (see Toda, 1979). It is in the section entitled 'Snakes on the Move'. This article deals with toy 'paper snakes' and explains the mechanism of a cornice. It is taken that since snakes advance by extending and contracting, they must bend

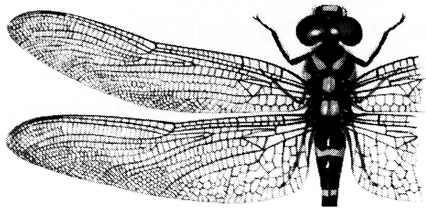


Figure 3. The wing base of a dragonfly (from 'Picture Book of Creepy Crawlies', 1987)

themselves in a zig-zag similar to the Miura folding.

Thinking along those lines, the cornices in Chinese lanterns and cameras all zig-zag in the same way. Isn't it true that right angles are no good for folding up nicely like this? It is thought that the blood vessels in the bases of dragonfly and butterfly wings are not orthogonal. Perhaps when resting with the wings closed, right angles would prevent the wings from being neatly folded away. Figure 3 shows the base of Cordulegasteridae wings ('Picture Book of Creepy Crawlies', 1987). The anterior edge of the wing base is zig-zagged like a Miura folding. The pattern of blood vessels is also parallel, and it is complex with few right-angled components visible. The dragonfly develops from a larva, metamorphoses into an adult insect, and the wings open from a closed condition, so there is some relationship with Miura folding. The straight lines we learn about in mathematics, circles, 2nd order functions, as well as curves and so on are simple because they are artificial. These kinds of curves rarely exist in the natural world. Perhaps there's a reason for the complex patterns? The progression up to the present day must certainly have required a great many years.

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