The Mysterious Number 6174 – One of 30 Amazing Mathematical Topics in Daily Life

Yutaka Nishiyama

GENDAI SUGAKUSHA, CO., 2013, 322 PAGES
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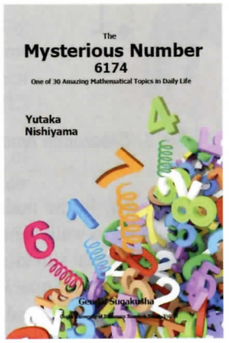
This book is part of the Osaka University of Economics Research Series, Vol. 79. The title is designed for a wide range of readers, even those who dislike mathematics.



It is a collection of 30 self-contained articles on topics in recreational and everyday mathematics. These are well balanced and range from curiosities in puzzles with numbers and shapes to more practical results related to science and engineering.

The title refers to one of these – 6174 is Kaprekar's Number, the result of a simple number puzzle. Take any four-digit number: put the digits in high-to-low order, then in low-to-high order. The difference between these two numbers is calculated. Repeat the process, and eventually we will reach 6174. For example:

$$4321 - 1234 = 3087$$
$$8730 - 0378 = 8352$$
$$8532 - 2358 = 6174.$$

The chapter on Kaprekar's Number states that the operation can be applied to recurring decimals – but this generalisation is not well explained. The operation is also dependent on the base 10 representation of the numbers – some extension to other bases would be interesting. There is some investigation into corresponding results for numbers with fewer or more than four digits.

Many apparently simple problems conceal profound results in unexpected branches of mathematics. Chapter 4: Stairway Light Switches starts with a mundane practical question and takes the reader on a journey through Boolean Algebra, truth tables and logic circuits leading to a hypothetical 'infinite story house' reminiscent of Hilbert's hotel.

Inspiration is drawn from physics (boomerangs, fans and solar panels), engineering (Oldham's coupling and air conditioners, building blocks and harmonic series) and biology (egg shapes and numbers of petals on a flower) as well as numbers (calculation of $\sqrt{2}$ and π) and geometry.

Unexpected results in probability and an apparent change in area when pieces of a rectangle are rearranged will be familiar to seasoned mathematical puzzlers however the explanations do reveal fresh insight into our perceptions. Some of the mathematics is actually quite advanced, for example Chapter 23 on the Brachistochrone curve (the problem of quickest descent) uses the calculus of variations and the Euler-Lagrange equation.

The last few chapters tackle some cultural issues including differences between British and Japanese attitudes to the Sudoku puzzle. The author refers to 'The Healthy Playful Psychology of the British' (p. 292) in particular the observation that the study of Sudoku in the UK is regarded as 'legitimate mathematics'.

The text is well-referenced but there is no index, which presents some minor problems for a reviewer. The author's aim is to show 'just how wonderful maths really is' and this is more than achieved. Some chapters will certainly appeal even to those who dislike mathematics but need to use it, such as undergraduates of other subjects – it is very readable and well-illustrated, revealing the importance of mathematics in daily life. In other chapters the material is quite demanding but the lucid explanations will be appreciated by the experienced mathematician. Overall there is something for readers at all levels, and I would recommend it to anyone with even a slight curiosity about mathematics.

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The Signal and the Noise: The Art and Science of Prediction

Nate Silver

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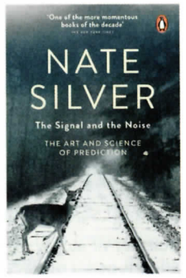
Nate Silver introduces *The Signal and the Noise*, as 'a study of why some predictions succeed and why some fail'. The areas covered include earthquakes, baseball, terrorism, global warming, politics, chess, poker, weather, the housing bubble and the spread of the flu virus. He builds on success from predicting the winner of the last US Presidential election in 49 out of 50 states. The book is very well researched and commentary is derived from interviews with specialists from each area of study. Silver introduces the material on a case study basis, allowing the reader to draw their own inference. He discusses characteristics of good forecasting, 'thinking probabilistically', factoring in uncertainty, updating and refreshing a model when new information becomes available and taking aggregate forecasts. He discusses building in flexibility, when he designed a system to predict the statistics of Major League Baseball players, '– its forecasts were probabilistic, for instance, outlining a range of possible outcomes for each player'.



Silver contrasts where forecasting is easier in some areas than others, comparing baseball and economics. The recording of baseball statistics over some 140 years, meaning a hypothesis can be effectively tested to a high level of statistical satisfaction in comparison to the complications involved in obtaining and forecasting from economic data. Silver expands on these difficulties: 'Most statistical models are built on the notion that there are independent variables and dependent variables, inputs and outputs, and they can be kept pretty much separate from one another. When it comes to the economy, they are all lumped together in one hot mess'. The difficulty of prediction is further illustrated when unilateral decisions are made; the Saudi Arabian oil minister in 2014 deciding not to cut oil production to keep supply and demand in equilibrium resulted in a significant price fall.

A strong element of the book is his consideration of underlying theoretical understanding of the ability to effectively forecast and to developing forecasting potential. A case in point is weather forecasting, which has developed over the past 10 to 20 years, assisted by improved computer power. Forecasters have an understanding of weather to the molecular level, however concede that in earthquake prediction, 'Most events occur at a depth of 15km underground. We don't have a hope of drilling down there, …. There's no way to directly measure the stress'. Silver discusses the power law distribution that can govern the frequency of earthquakes. On a wider level he advocates Bayes theorem as appropriate to high levels of uncertainty in decision making, and gives a clear description of its application to global warming, terrorism and bubbles in financial markets. He illustrates this conditional probability theorem refining the model as new information is realised. Silver addresses the human drivers associated with forecasting: US weather stations over-predict rain as they know it is the one error that the public will not forgive, 'who cares if there is a little wet bias, especially if it makes for better television?'; credit rating agencies motivation for validating mortgage products contributing to the housing bubble and economic forecasters not giving their forecasts within percentage intervals so as not to appear lacking in confidence, are some examples.

The Signal and the Noise is an impressive contribution to the study and practise of prediction.

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Quantum Physics and Linguistics

Edited by Chris Heunen, Mehrnoosh Sadrzadeh and Edward Grefenstette

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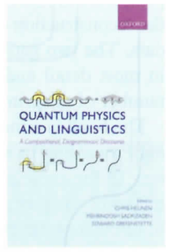
This book presents the idea that large amounts of quantum mechanics is concerned with compositional reasoning about information flows between the sub systems of a quantum mechanical system and how these flows contribute to the properties of the overall system. Similarly a significant proportion of linguistics is also concerned with compositional reasoning. The book considers the mathematics that lies behind compositionality and how these methods can be applied to reasoning about the phenomena that apply to both areas of knowledge.



For some years now there has been an effort to reframe quantum mechanics from the perspective of mathematical category theory rather than vector spaces. This has led to a significant simplification in the portrayal of aspects of quantum information theory. Linguistics has also been re-considered from the viewpoint of category theory. Thus it has become appropriate to consider similarities and differences between the two, apparently unrelated areas of study.

The book consists of thirteen chapters. The first chapter introduces the use of category theory in quantum physics and also computer science. Chapters 2–7 consider different types of category and the ways in which a particular type of category can be applied to a specific area of quantum physics, perhaps initiating different areas of study. Finally, Chapters 8–13 turn to the application of categories and use of compositional reasoning in linguistics. Here the authors describe how various types of category can be used to analyse sentences and meaning. A description of the chapters follows.

Chapter 1 – Introduces and discusses the idea that there are different types of structure that can be used to analyse mathematics.

Chapter 2 – Introduces Frobenius algebras with particular emphasis on their relation to Hopf algebras, using graphical notation. These algebras have been applied both to mathematics and physics but also to linguistic analysis.

Chapter 3 – Uses symmetric monoidal categories to reformulate quantum mechanics from the viewpoint of the underlying algebra rather than directly using Hilbert spaces. This greatly simplifies the analysis of quantum information theory.

Chapter 4 – Introduces quantum groups and uses Hopf algebras in braided categories. Braided categories are introduced in the chapter.

Chapter 5 – Starts with reminders about Hopf algebras, monoidal categories and the Tannaka reconstruction theorem and finally shows how variations of Hopf algebras are related to variations of the Tannaka theorem.

Chapter 6 – Introduces modular categories, these are braided tensor categories that are linear over a field.

Chapter 7 – This is the final chapter of the book that deals with quantum physics. It starts by discussing scalars from the viewpoint of categories and continues with some material on monads and Lawvere theories. From there additive monads and refined triangle with involutions are introduced.

Chapter 8 – Initiates the linguistic part of the book. Recently there have been efforts in linguistics to extend linguistic models of meaning from words to sentences. This uses a Type system that includes an idea of evaluation. The Type system is based on a categorical grammar. The chapter continues to consider whether the evaluation system is reversible, i.e. information lost, and concludes that, generally, information is lost.

Chapter 9 – This chapter relates two semantic representations of language: functional logical models and the distributional vector models. The chapter suggests a means of accounting for the logical content of 'noise' and transforms the compositional extensional representation into a conceptual representation, both former and latter being based on closed categories.

Chapter 10 – In the late 1950s/early 1960s Lambek introduced a calculus for deciding if an expression is well formed. More recently there have been extensions to his ideas investigating using this approach in natural languages that are beyond the context free languages studied previously. This chapter introduces the use of proof nets for analysing the meaning of sentences.

Chapter 11 – Uses a vector space approach within which logical semantics can be embedded. The work here is not completely new but the mathematics has been made tidier with the application of some new ideas.

Chapter 12 – This chapter takes an historical approach to distributional models of meaning and considers the ways in which the models have changed, going on to recent moves to include compositionality.

Chapter 13 – An actual compositional distributional model of meaning is outlined, specifically the categorical compositional distributional model of meaning developed by Clark, Coecke and Sadrzadeh.

This book will be of interest to people with a background in category theory, whether from a quantum physics background or linguistic background. It may also be useful to researchers who use the approach presented here to apply categorical ideas and compositional reasoning to another field of knowledge, resulting in further links being found between apparently unrelated fields.

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